CONSTRAINING THE MAGNETIC EFFECTS ON H I ROTATION CURVES AND THE NEED FOR DARK HALOS

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ABSTRACT

The density profiles of dark halos are usually inferred from the rotation curves of disk galaxies based on the assumption that the gas is a good tracer of the gravitational potential of the galaxies. Some authors have suggested that magnetic pinching forces could alter significantly the rotation curves of spiral galaxies. In contrast to other studies which have concentrated in the vertical structure of the disk, here we focus on the problem of magnetic confinement in the radial direction to bound the magnetic effects on the H I rotation curves. It is shown that azimuthal magnetic fields can hardly speed up the H I disk of galaxies as a whole. In fact, based on virial constraints we show that the contribution of galactic magnetic fields to the rotation curves cannot be larger than ~ 10 km s^{-1} at the outermost point of H I detection, if the galaxies did not contain dark matter at all, and up to 20 km s^{-1} in the conventional dark halo scenario. The procedure to estimate the maximum effect of magnetic fields is general and applicable to any particular galaxy disk. The inclusion of the surface terms, namely the intergalactic (thermal, magnetic or ram) pressure, does not change our conclusions. Other problems related with the magnetic alternative to dark halos are highlighted. The relevance of magnetic fields in the cuspy problem of dark halos is also discussed.

Subject headings: galaxies: halos — galaxies: kinematics and dynamics — galaxies: magnetic fields — galaxies: spiral

1. INTRODUCTION

The rotation curves of spiral galaxies, especially dwarf galaxies and low surface brightness galaxies, are used to derive the profiles of the dark halos which are very useful as tests for the standard cold dark matter scenario (e.g., de Blok & Bosma 2002). The usual assumption is that the dynamics of the neutral atomic gas is a good tracer of the radial gravitational force. It can be shown that although spiral waves and non-circular motions may contribute to produce substructure in the rotation curves (e.g., de Blok, Bosma & McGaugh 2003), they do not alter the rotation curves significantly for the galaxies selected in these investigations. Galactic magnetic fields could also affect the gas dynamics in spiral galaxies (Piddington 1964; Peratt 1986; Battaner et al. 1992; Sánchez-Salcedo 1997; Benjamin 2000; Beck 2002, 2003).

The random small-scale component of the galactic magnetic fields acts as a pressure, giving support to the disk and, therefore, leading to a rotation (slightly) slower than the gravitational circular speed. By contrast, some configurations of the large-scale magnetic field can give rise to a faster circular velocity. Nelson (1988) and Battaner et al. (1992) proposed the so-called "magnetic alternative" to dark matter in which the contribution of the magnetic pinch could be large enough to explain the observed rotation curves of spiral galaxies without the necessity of dark matter. This is an interesting possibility because some universal properties such as the disk-halo conspiracy or the H I-dark matter constant relation, first noticed

by Bosma (1978), would have a natural explanation in this scenario (Sánchez-Salcedo 1996a), as well as the truncation of stellar disks (Battaner, Florido, Jiménez-Vicente 2002). However, after the publication of the paper by Battaner et al. (1992), a plethora of possible difficulties faced by the model was pointed out by different authors (Cuddeford & Binney 1993; Jokipii & Levy 1993; Persic & Salucci 1993; Vallée 1994; Katz 1994; Pfenniger, Combes, & Martinet 1994; Sánchez-Salcedo 1996b, 1997). In an extensive review, Battaner & Florido (2000) argue that all the shortcomings can be overcome and, therefore, magnetism can explain the way in which galaxies rotate, without the help of dark matter.

A certain MHD configuration has the ability of producing a faster rotation of the whole plasma in the system, i.e. magnetic confinement, when the net contribution of the magnetic fields to the virial theorem is negative (in the conventional notation). In an isolated system, it is well known that the magnetic field contribution to the virial theorem, applied to a large enough volume, is *positive or zero*, but never negative, reflecting the net expansive tendency of magnetic fields (e.g., Shafranov 1966). Jokipii & Levy (1993) used this argument to conclude that in the presence of magnetic fields, galaxies would need even more dark matter than in the unmagnetized case. One could argue that, since the virial theorem uses global (volume-integrated) variables, it is possible that magnetism has a "centrifugal" action in the vertical direction but centripetal in the radial direction (radial confinement) and,

therefore, the expansive tendency of magnetic fields manifests only in the vertical direction. We advance that this claim is very wrong. An alternative way out is to assume that galaxies are not at isolation but embedded in the ubiquitous (magnetized) intergalactic medium which would be responsible for the required radial confinement.

Motivated by these ideas and given the importance of a correct interpretation of the rotation curves for Cosmology, we will try to put bounds on the effects of magnetic fields on the H I rotation curves. However, we do not restrict ourselves to discuss the viability of explaining rotation curves without dark halos; the orbits of Galactic globular clusters, the satellite galaxies and possibly the dynamics of remote stars in the outer Galactic halo, which are not affected by magnetic fields, strongly argue in favor of a dark halo around the Milky Way (e.g. Freeman 1997). The main issue is to see whether galactic magnetic fields can alter appreciably the H I rotation curves, especially in external galaxies for which the dark matter content is almost entirely derived from the rotation curve of the gas. Only if the answer is negative, we can be confident that the profiles of dark halos, the dark matter content in galaxies as well as other interesting empirical correlations found between, for instance, halo parameters and luminosity are reliable.

Our investigation will be focus on the *radial* equilibrium configuration of magnetized rotating disks. For reasons that should become clear later, the analysis of the radial equation of motion puts stringent bounds on the maximum magnetic effects to the rotation curves; this being free of other complicating assumptions that appear in studies of the vertical equilibrium configuration such as the external pressure, the vertical dependence of the magnetic field within the height of the H I thin disk, or the flattening of the dark halo, which are all very uncertain.

The paper is organized as follows. In §2 we describe the ingredients of a simplified model in which magnetic fields could give rise to a faster rotation in the outer parts of H I disks. In §3 the maximum effect on the rotation curves is studied. Other difficulties inherent to the magnetic alternative are discussed in §4. The case of magnetic fields producing a slower rotation and their role as a possible remedy to the cuspy problem of dark halos is left to §5 and the final remarks are given in §6.

2. INGREDIENTS OF THE MAGNETIC MODEL AND BASIC THEORY

In this Section, we describe some basic concepts useful for understanding the remainder of the paper. The issue of the problem of radial confinement will be addressed in §3.

2.1. *The One-dimensional Case*

In presenting the role of the magnetic tension in the H I rotation curves, the equations are simplified by assuming an equilibrium, steady-state, axisymmetric disk of a slightly ionized gas with a purely azimuthal large-scale magnetic field B_ϕ (e.g., Battaner et al. 1992; Sánchez-Salcedo 1997). The radial component of the equation of motion of the gas at $z = 0$ in cylindrical coordinates reads

$$v_\phi^2(R) = R\frac{d\Phi}{dR} + \frac{R}{\rho}\frac{dP}{dR} + \frac{1}{8\pi R\rho}\frac{d(R^2 B_\phi^2)}{dR}, \qquad (1)$$

where $\rho(R)$ is the gas density at $z = 0$, $v_\phi(R)$ the circular speed of the gas, $\Phi(R)$ the gravitational potential created by all masses in the system, and $P(R)$ is the gas pressure consisting of the thermal, turbulent, convective, and cosmic ray components plus also the magnetic pressure arising from the random magnetic field component, but does not include the magnetic pressure arising from the large-scale magnetic field. All these variables should be taken at the midplane of the disk. In equation (1) it was assumed $v_\phi \gg v_R, v_z$, and thus we kept only the dominant terms associated with v_ϕ. In terms of the total pressure $P_{\text{tot}}(R) = P(R) + B_\phi^2(R)/8\pi$, it can be rewritten as:

$$v_\phi^2(R) = R\frac{d\Phi}{dR} + \frac{R}{\rho}\frac{dP_{\text{tot}}}{dR} + \frac{B_\phi^2}{4\pi\rho}. \qquad (2)$$

The second term of the RHS is the radial pressure force that acts inwards or outwards depending on whether P_{tot} increases outwards or inwards. The third term is the magnetic tensile force which always imposes a net inward force.

We should remind that ordered galactic-scale magnetic fields are not expected to be a general feature in all disk galaxies, especially in dwarf galaxies. Although observations of magnetic fields in these galaxies are scarce, they do not indicate magnetic fields with a global symmetry, but a much more chaotic galactic-scale field than in normal spirals (e.g., Chyzy et al. 2000). Therefore, models based on a regular azimuthal magnetic field may not be an adequate representation for all of them. In fact, indirect observations suggest that regular magnetic fields may be ignorable, at least in some dwarf galaxies. The dominance of spherical holes in the H I of the nearby dwarf galaxy IC 2574 has lead Walter & Brinks (1999) to conclude that large-scale magnetic fields are dynamically unimportant in the outer parts of this galaxy.

As it is shown in Appendix A, $P(R)$ is always a decreasing function of R in approximately-isothermal, exponential galactic disks, even if the disk were embedded in an ambient medium with high pressure. Therefore, the asymmetric drift always produces a force pointing outwards and hence it can never contribute to enhance the circular speed of the gas. Moreover, the asymmetric drift is almost negligible in rotating galactic disks except in very low-mass galaxies such as DDO 50 (Carignan, Beaulieu & Freeman 1990; de Blok & Bosma 2002; Bureau & Carignan 2002). Thus we will also ignore it, hence $dP_{\text{tot}}/dR \approx (1/8\pi)dB_\phi^2/dR$, and the force balance equation is simplified to

$$v_\phi^2(R) = v_c^2(R) + \frac{1}{8\pi R\rho}\frac{d(R^2 B_\phi^2)}{dR}, \qquad (3)$$

where $v_c^2 \equiv Rd\Phi/dR$ is the gravitational circular velocity. Note that the magnetic pressure arising from the large-scale field is kept in our analysis because it cannot be neglected in general. The role of the magnetic pressure term associated with the random (small-scale) magnetic field component is discussed in detail in §5. It should be also clear that the vertical gradient of P is not negligible when considering the vertical configuration of the disk, since it is the main agent responsible for the vertical support against gravity.

From Eq. (3) we see that for a purely azimuthal magnetic field decaying radially not faster than R^{-1}, the magnetic tension force produces radial confinement. Such configuration is usually called a linear or Z-pinch. The resulting inward magnetic force, the pinch, accounts for the required centripetal force responsible for an anomalous faster rotation. Our aim is to constrain the maximum effect of the Z-pinch in galactic disks from very general grounds. It is worth mentioning that the Z-pinch configuration is not used in tokamaks as interchange and other instabilities quickly destroy the magnetic configuration (e.g., Roberts 1967). Restricted to purely azimuthal fields, the force-free magnetic configuration occurs for a field profile $B_\phi = B_0(R_0/R)^2$, where B_0 is the magnetic strength at some radius R_0. For that profile, the outward magnetic pressure force is in balance with the inward tensile force at any radius. This profile is, however, rather unphysical because it is singular at $R = 0$.

Observations of galactic magnetic fields suggest that the radial component of the field, B_R, is comparable to B_ϕ at least in the optical disk and hence neglecting B_R may not be a good approximation. One could argue, however, that the magnetic pitch angle becomes very small in the outermost parts of galactic disks, where measurements of the field orientation are scarce. Since B_R is not well-suited to produce radial confinement, the most optimistic and simplest situation, in order to maximize the magnetic effects on the rotation curves, occurs when the field is taken as purely azimuthal. As we are interested in putting upper limits on the role of magnetic fields we will assume throughout this paper that $B_R = 0$ in the outer disk to give the model as much leeway as possible, as it is assumed in the magnetic alternative.

An important contribution to P is the turbulent pressure. For a certain component of the interstellar medium, the averaged turbulent pressure is given by $\rho v_t^2/2$, where v_t is the one-dimensional turbulent velocity dispersion. For the H I components, $v_t \approx 6$–10 km s^{-1} (e.g., Boulares & Cox 1990) and is nearly constant with galactocentric radius and also from galaxy to galaxy (Sellwood & Balbus 1999, and references therein). The magnetic field is said to be in equipartition with the turbulent motions when the gas-to-magnetic field pressure ratio is 1, or equivalently, when the turbulent velocity dispersion coincides with the Alfvèn speed. In the literature, one often finds the statement that dynamo action in a thin disk saturates at equipartition field strengths (e.g., Ruzmaikin, Sokoloff & Shukurov 1988; Shu 1992; Poezd, Shukurov & Sokoloff 1993; Beck et al. 1996; Kulsrud 1999). Therefore, it is interesting to estimate those values of $v_\phi - v_c$ for which the large-scale magnetic field would be in superequipartition; this is done in the next. Denoting by v_A the Alfvèn velocity associated with the regular field, we can write $v_\phi^2 - v_c^2 \equiv (8\pi R\rho)^{-1}d(R^2 B_\phi^2)/dR = v_A^2 + (R/8\pi\rho)dB_\phi^2/dR$. In the likely case that the strength of the large-scale magnetic field does not increase with galactocentric radius it implies $v_\phi^2 - v_c^2 \lesssim v_A^2$. Hence, for changes in the rotation velocity of $v_\phi^2 - v_c^2 > v_t^2$, the regular magnetic field should be above equipartition with the turbulent motions. The above condition is equivalent to $v_\phi > \sqrt{v_t^2 + v_c^2}$ or, in terms of $v_\phi - v_c$:

$$v_\phi - v_c > (v_t^2 + v_c^2)^{1/2} - v_c \approx v_c\left(1 + \frac{1}{2}\frac{v_t^2}{v_c^2}\right) - v_c = \frac{1}{2}\frac{v_t^2}{v_c}. \tag{4}$$

For $v_c \approx 100$ km s^{-1}, variations of $v_\phi - v_c \gtrsim 0.5$ km s^{-1} already require magnetic fields over equipartition. Such a situation is not very appealing to most dynamo theorists. However, in order to present our results as generic as possible and since there are some observations that could suggest magnetic energy densities above the turbulent ones in the disk of galaxies (e.g., Beck 2002, 2003) and in the Galactic center (e.g., Moss & Shukurov 2001), we will not employ equipartition when discussing the magnetic effects on the rotation curves.

So far, the usual way to proceed was to integrate Eq. (1) out to a certain radius in order to derive the required radial profile for $B_\phi(R)$ to explain the observed rotation curve v_ϕ, given the volume density at $z = 0$ and the gravitational circular velocity (including a contribution of the dark halo as in Sánchez-Salcedo (1997) or only the luminous components as in Battaner et al. (1992)). The question that arises is whether these local solutions can be matched with boundary conditions at infinity or not. This issue is very delicate and it will be proven very useful to put bounds on the influence of galactic magnetic fields in the gas dynamics at large scales. Before that, however, it is illustrative to describe how galaxies can reach a configuration with a faster rotation in order to have a complete picture of the underlying physics. This is briefly described in the next subsection.

2.2. *The Phase of Magnetic Contraction*

The assumption that the magnetic field is mainly azimuthal immediately suggests that it commenced to grow after the galactic disk has formed. In the absence of magnetic tension, the *initial* galactic disk or protodisk should have a rotation speed given by the mass distribution $v_\phi^2 = v_c^2 = Rd\Phi/dR$. If the rotation curves are significantly affected by the magnetic field, one should expect that, as the magnetic field is amplified no matter by what process, the disk initially in gravitational force balance, will experience a radial contraction caused by the growing magnetic tension, leading to a disk with a faster rotation due to angular momentum conservation. We will refer to this process as the magnetic contraction phase (MCP). If the mass loss by outflows is unimportant, the MCP is the only way capable of spinning up the disk because the total angular momentum stored in the fields ($\propto \int \boldsymbol{r} \times [\boldsymbol{E} \times \boldsymbol{B}]\, dV$) is negligible in both the initial protodisk and in the final configuration. Other processes, such as turbulent viscosity, are very inefficient to produce the desired effect.

The conservation of angular momentum implies that, for a galaxy similar to the Milky Way with a dark halo and a rotation speed of 220 km s^{-1} at 30 kpc, an increase of 10 km s^{-1} in the velocity speed requires a radial contraction of 1.3 kpc at that radius. The contraction must be huge in the magnetic alternative; see §4.1.

3. GALACTIC DISKS WITH FASTER ROTATION. THE PROBLEM OF THE RADIAL CONFINEMENT

3.1. *Isolated Galaxies*

In the previous section it was pointed out that there is a pinching effect only if B_ϕ does not decrease faster than $1/R$. However, this decay is untenable at large R in real astrophysical systems as it will be shown in the next and, therefore, it is inevitable that beyond a certain radius, the magnetic tension produces a force pointing outwards (see Fig. 1 for a visualization). In fact, the slowest radial magnetic decay occurs in the ideal case of having strict cylindrical geometry with currents, J_z, flowing from $z = -\infty$ to $z = \infty$ (otherwise, if currents are confined to a certain region, $\boldsymbol{B}$ must decrease with radius at least as fast as a dipole field for large $\boldsymbol{r}$: $B \leq K/r^3$, where K is a constant). Under strict cylindrical geometry [1], the condition of closed current loops is equivalent to imposing $\int_0^R J_z R' \, dR' \to 0$ at large R (hereafter, this will be our definition of *isolated system* in a cylindrical system). From Ampère's law, this requirement implies that $B_\phi < C/R$ at large R, being C a constant. So there must exist a certain radius denoted by R_{kep} at which the magnetic field starts to decay as $1/R$ or faster. R_{kep} is therefore the radius at which the rotation speed of the gas coincides with the local Keplerian velocity. If R_{HI} is the radius of the last point of H I detection, we require $R_{\mathrm{kep}} > R_{\mathrm{HI}}$ in order to have an anomalous faster rotation along the observed disk.

The system will be in a configuration of equilibrium at $R > R_{\mathrm{kep}}$ only if the inward gravitational force is higher than the outward magnetic force

$$R\frac{d\Phi}{dR} > -\frac{1}{8\pi R\rho}\frac{d(R^2 B_\phi^2)}{dR}. \qquad (5)$$

If this condition is not fulfilled the disk will expand radially due to the magnetic forces. This condition can be written as

$$B_\phi(R) > \frac{1}{R}\left[R_{\mathrm{kep}}^2 B_{\phi,\mathrm{kep}}^2 - 8\pi \int_{R_{\mathrm{kep}}}^R \rho\, R\, v_{\mathrm{c}}^2 \, dR \right]^{1/2}, \quad (6)$$

where $B_{\phi,\mathrm{kep}}$ is the magnetic field strength at R_{kep}. Since a dependence $B_\phi \approx C/R$ at large R is unphysical (see above), an upper limit to $B_{\phi,\mathrm{kep}}$ can be established from Eq. (6):

$$B_{\phi,\mathrm{kep}}^2 \leq \frac{8\pi}{R_{\mathrm{kep}}^2} \int_{R_{\mathrm{kep}}}^{R_{B_\phi}} \rho(R,0)\, R\, v_{\mathrm{c}}^2(R) \, dR, \qquad (7)$$

where R_{B_ϕ} is the radius where $B_\phi \approx 0$. In practice we will take $R_{B_\phi} \to \infty$ to estimate the maximum contribution. The maximum value for $B_{\phi,\mathrm{kep}}$ corresponds to a situation in which there is balance at $R > R_{\mathrm{kep}}$ between gravitational and magnetic forces and thus the material outside the R_{kep} circle has no rotation at all.

The upper limit derived above has an easy interpretation. In principle, one could achieve very large differences $v_\phi - v_{\mathrm{c}}$ if one would make the density arbitrarily small than the Alfvèn speed would be arbitrarily large. However, according to Eq. (5), this is not possible globally because the magnetic field has to be held in by the radial weight of the plasma in the radial gravitational field since the net effect of magnetic fields is never confining.

It is customary and natural to discuss this kind of constraints using the virial theorem which says that the net nature of magnetic fields is expansive or null (but never confining) in both the radial and vertical directions (Appendix B). More specifically, for toroidal magnetic fields and strict cylindrical geometry, the total rotational energy of an isolated system should be the same with or without magnetic fields, keeping the mass distribution fixed (so that $\Phi(R)$; see Appendix B for further details). By imposing that constraint, a similar upper limit for B_{kep} is found (Appendix C), except by a factor ($\lambda_{\rho B}$), between 1–2, that takes into account possible three-dimensional effects.

It is convenient to consider first the extreme and hypothetical case that spiral galaxies do not host dark matter halos. For a galaxy with a mass in gas and stars M, $\Phi(R) \approx -GM/R$ in the outer parts and

$$B_{\phi,\mathrm{kep}}^2 \leq B_{\mathrm{cr}}^2 \equiv 8\pi\lambda_{\rho B}\frac{GM}{R_{\mathrm{kep}}^2} \int_{R_{\mathrm{kep}}}^\infty \rho(R,0)\, dR. \qquad (8)$$

The assumption that $\Phi(R) \approx -GM/R$ is satisfactory for many high surface density galaxies (e.g., van Albada et al. 1985) but it is not necessarily fulfilled in some low surface brightness galaxies (LSB) and dwarfs for which the gas surface density is not exponential and its contribution to the potential may be comparable or even higher than that from the stellar component at radii $\sim R_{\mathrm{HI}}$ for certain values of the adopted stellar mass-to-luminosity ratio. However, we are confident that our conclusions would not change for these galaxies even adopting the most unfavorable assumptions.

In the optimistic and rather unlikely situation that B_ϕ were constant with R out to R_{kep}, i.e., $B_\phi(R) = B_{\mathrm{cr}}$ at $R < R_{\mathrm{kep}}$, the characteristic variation $v_\phi^2 - v_{\mathrm{c}}^2$ would be $v_\phi^2 - v_{\mathrm{c}}^2 \approx B_{\mathrm{cr}}^2/(4\pi\rho)$. By using Eq. (8), it is possible to estimate the maximum relative variation of the circular speed of the gas at a radius $R < R_{\mathrm{kep}}$:

$$\left.\frac{v_\phi^2 - v_{\mathrm{c}}^2}{v_{\mathrm{c}}^2}\right|_R = 2\lambda_{\rho B}\left(\frac{R}{R_{\mathrm{kep}}}\right)\left(\frac{R_{\mathrm{g}}}{R_{\mathrm{kep}}}\right)\exp\left(\frac{R - R_{\mathrm{kep}}}{R_{\mathrm{g}}}\right), \tag{9}$$

where we have adopted a radial exponential decay for $\rho(R,0)$ with scale R_{g} (e.g., Bland-Hawthorn, Freeman & Quinn 1997). Note that $(v_\phi^2 - v_{\mathrm{c}}^2)/v_{\mathrm{c}}^2$ is not sensitive to P_{ext} because even though the density at the midplane increases with P_{ext}, equation (9) does not depend on the absolute value of the density but only on R_{g}. The main source of uncertainty stems from the value of R_{kep}, but the most optimistic situation occurs when $R_{\mathrm{kep}} \approx R_{\mathrm{HI}}$, so we will assume for the discussion that both radii are identical, ≈ 30 kpc for common spiral galaxies. In order to estimate $v_\phi^2 - v_{\mathrm{c}}^2$ from Eq. (9), we will assume that the mass of gas in the disk beyond R_{HI} and within $|z| < h_1$, being h_1 the characteristic scale height of the gas disk (see Appendix C for further details), is 0.5 percent the total mass of the disk. This seems to be the case of dwarf galaxies (van den Bosch et al. 2001), but it is probably a too generous value for normal spiral galaxies. The latter assumption corresponds to a radial scale length for the gas, R_{g}, of ~ 5.5 kpc. Putting these values into Eq. (9), we obtain $(v_\phi^2 - v_{\mathrm{c}}^2)/v_{\mathrm{c}}^2 = 0.36\lambda_{\rho B}$ at $R = 30$ kpc. If we take for

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[1] In such a situation, the magnetic field is always purely azimuthal, consistent with the assumptions given in §2 and, hence, the essential physics of the mechanism giving rise to greater azimuthal velocities of the H I disk, is entirely contained within the one-dimensional radial models.

reference values $v_{\rm c} = 100$ km s^{-1} at 30 kpc, the maximum change in the rotation velocity, $\Delta v \equiv v_\phi - v_{\rm c}$, ranges between 15 to 30 km s^{-1} for $\lambda_{\rho B} = 1$ to 2, respectively.

This value for Δv should be regarded as a generous estimate for two reasons. First, the azimuthal field is not expected to be approximately constant while the volume density of gas at $z = 0$ varies by a factor of ~ 50–100 (Diplas & Savage 1991). Secondly, the adopted equilibrium configuration, namely $v_\phi(R) = 0$ at any $R > R_{\rm kep}$, cannot be reached in practice, starting from a rotating disk in equilibrium with the gravitational force, due to the conservation of angular momentum. In fact, the magnetic field begins to grow in the disk as discussed in §2.2, creating a magnetic force directed outward beyond $R_{\rm kep}$. This region will undergo a continuous outward drift, while the inner disk ($R < R_{\rm kep}$) must be subjected to the required contraction phase, generating a gap in the surface density of the disk. In order to quantify this effect, we assume for simplicity that the outer disk ($R > R_{\rm kep}$) suffers an approximately self-similar expansion with factor $\psi > 1$, i.e., $\rho(R) = (\rho_0/\psi^2)\exp(-R/\psi R_{\rm g})$ at $R > \psi R_{\rm kep}$, in the final equilibrium configuration. From the conservation of angular momentum, it can be shown that the right-hand side of Eq. (9) is now a factor $\psi^2/(\psi - 1)$ smaller (Appendix C). Hence, the optimized situation occurs for $\psi = 2$ and then $(v_\phi^2 - v_{\rm c}^2)/v_{\rm c}^2 < 0.1\lambda_{\rho B}$ at $R = 30$ kpc. This implies that $\Delta v_\phi < 10$ km s^{-1} at the periphery of H I disks in normal spiral galaxies under the assumption that they contained only stars and the observed gas. Therefore, it is not possible to banish dark halos by magnetic fields because one would require much larger values of Δv_ϕ.

For disk galaxies with a dark halo, as postulated in the conventional scenario, it is possible to follow the same procedure to estimate the maximum effect of magnetic fields when $v_{\rm c} = $cte, instead of $v_{\rm c}^2 = GM/R$ (see Appendix C). Similar arguments suggest that the expected local, and probably transient, variations in the rotation speed larger than ~ 20 km s^{-1} are very unlikely even in models including dark halos (see also §2.2 and Sánchez-Salcedo 1997).

In the next subsections, it will be shown that these upper limits for Δv_ϕ also apply even when galaxies are not isolated systems but are embedded in an ambient medium, thereby making these estimates very robust.

3.2. *Non-Isolated Galaxies*

It could be argued that the model described by Eq. (1) is too simplistic because other terms not considered in that equation, as a radial flow or other components of the magnetic field, might be important, regarding the radial confinement, beyond $R_{\rm HI}$[2]. In fact, unlike the radial component of the field, the z-component may contribute to the radial confinement of the disk and will be included in our analysis. In addition, the hydrodynamical pressure by a radial inflow due to intergalactic accretion of matter may also play a role. Since systems embedded in an ambient medium usually present sharp boundaries, it is convenient to use an integral form of the equations as that given by the virial theorem.

The scalar virial theorem for fields with $B_R = 0$ and strict cylindrical symmetry (i.e., $\partial/\partial z = \partial/\partial\phi = 0$) applied to a finite cylindrical volume V, bounded by a cylinder S of height L, can be written as:

$$2\mathcal{T} + 2\int_V P\,dV - 2P_{\rm S}V + \mathcal{W} + \mathcal{M}_{\rm cyl} - \int_S \rho\,(\boldsymbol{v}\cdot\boldsymbol{r})(\boldsymbol{v}\cdot d\boldsymbol{S}) = \frac{1}{2}\ddot{I}, \quad (10)$$

where $\mathcal{T}$ is the kinetic energy of the gas, $\mathcal{W}$ its gravitational energy in the gravitational field created by all masses in the system, P the gas pressure as defined in §2.1, $P_{\rm S}$ the gas pressure at the boundary surface, $\boldsymbol{r}$ is a coordinate vector, $\ddot{I} \equiv d^2I/dt^2$ with I the trace of the moment of inertia tensor and $\mathcal{M}_{\rm cyl}$ is given by

$$\mathcal{M}_{\rm cyl} = \frac{1}{4\pi}\left(\int_V B_z^2\,dV - \left(B_{z,{\rm S}}^2 + B_{\phi,{\rm S}}^2\right)V\right), \quad (11)$$

(e.g., Fiege & Pudritz 2000 and Appendix B). Throughout this paper the subscript S refers to quantities valuated at the radial surface of our cylindrical volume with radius $R_{\rm S}$, and the subscript 'cyl' stands for the cylindrical case. This form for the virial theorem is valid for any arbitrary helical field ($B_R = 0$) provided cylindrical geometry; no other additional assumption is required. The surface integral terms including the $z = \pm L/2$ surfaces are taken into account in Eq. (11). The integration of the magnetic terms can be found in Appendix B. For any further details in the derivation, we refer the reader to the paper by Fiege & Pudritz (2000).

We can identify two new vias of radial confinement: by ram pressure [last term of LHS of Eq. (10)] or by an external magnetic field. The ambient pressure ($P_{\rm S}$), on the other hand, only produces vertical confinement as discussed in Appendix A. If these new terms are able to produce radial confinement then the constraint given by Eq. (7) could be somehow relaxed. The possible role of these terms will be discussed in two separate sections.

3.2.1. *Ram Pressure*

So far, we have neglected the terms involving v_R, because H I observations show that v_R is very small compared to v_ϕ for the galaxies under consideration. Beyond the edge of the observed H I disk, the value of v_R is uncertain. We consider now the surface integral of the momentum stress in Eq. (10), which may have a negative sign for the case of strong mass accretion into the disk. In fact, it can be shown that even though $\ddot{I}$ is expected to be negative, the combination $-\ddot{I}/2 - \int_S \rho\,(\boldsymbol{v}\cdot\boldsymbol{r})(\boldsymbol{v}\cdot d\boldsymbol{S})$ could be still negative, showing the confining effect of the ram pressure. (For example, in the one-dimensional case of a supersonic flow that slows down to rest by crashing perpendicularly to a rigid plane wall and forms a strong adiabatic shock, $-\ddot{I}/2 - \int_S \rho\,(\boldsymbol{v}\cdot\boldsymbol{r})(\boldsymbol{v}\cdot d\boldsymbol{S}) = -(2/3)\int_S \rho\,(\boldsymbol{v}\cdot\boldsymbol{r})(\boldsymbol{v}\cdot d\boldsymbol{S})$.)

Since spherical accretion cannot confine a disk, the inflow should be extremely collimated parallel to the galactic plane, i.e. $v_R \gg v_z, v_\phi$, for which there is no evidence. Moreover, this collimation is very unlikely to occur if the disk forms from a sphere of gas. Gas cools and parcels of gas acquire circular orbits, conserving approximately angular momentum, when they collapse to settle into a disk, leading to $v_R \to 0$ (see Rögnvaldsson 1999 for results based on numerical simulations). Furthermore, in the hypothetical case that such inflow were possible by some unknown mechanism, the accretion speed should be so high that an

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[2] Within the $R_{\rm HI}$ circle, observations suggest that other terms are unimportant for those galaxies under consideration.

accretion shock would be unavoidable, transforming ram pressure into thermal pressure and probably generating a hydraulic jump in the edge of the disk and other observable features such as Hα emission. The hydraulic jump will divert the flow around the disk decreasing dramatically its confinement ability.

3.2.2. *Confinement by the Intergalactic Magnetic Field*

A coherent external (intergalactic) magnetic field could in principle contribute to the confinement under certain circumstances. It is important to distinguish here between the intracluster and the wider intergalactic field. As we are dealing with galaxies in the field, we are interested in the amplitude of the latter one. At present, setting observational constraints to the strength and topology of the intergalactic magnetic fields is a challeging task (see Ryu, Kang & Biermann 1988; Kronberg et al. 2001; and Widrow 2002 for a review) and, therefore, they are very uncertain. Coherent magnetic fields on scales much greater than the typical sizes of galaxies could be as large as $\sim 0.1~\mu$G in the optimistic situation that extra amplification through batteries, dynamos or other routes of magnetization were operative.

We assume the ideal situation that initially the pregalactic magnetic field is uniform and fills the protogalaxy making a finite angle α with the rotation axis of the galaxy. As the disk is forming by accretion of gas, the magnetic field lines are advected with the flow. It is well-known in studies of the collapse of interstellar clouds that, under flux-freezing conditions, this compression produces that the total magnetic contribution to the virial theorem is positive (see, e.g., Chapt. 24 of Shu 1992 and Chapt. 11 of Mestel 1999) and, therefore, the confinement by the external field is impeded. In order to have confinement, the disk must be able to get rid of the magnetic flux.

In the strict cylindrical symmetry, the external magnetic field must be vertical and then Eq. (11) with $B_{\phi S} = 0$ tells us that magnetic confinement is only possible if the volume-averaged vertical component of the magnetic field within V is strictly smaller than the magnetic field at the boundary. Since v_R is directed inward and $\partial v_R / \partial R < 0$ in the MCP, $|B_z|$ increases at every point of the disk due to the inward advection (see also Reyes-Ruiz & Stepinski 1996; Moss & Shukurov 2001); under those conditions confinement is not viable. This problem could be alleviated if magnetic field lines were expelled from the disk by ambipolar and turbulent diffusion. However, a thorough numerical study by Howard & Kulsrud (1997) shows that $|B_z|$ is also increased by ambipolar shearing terms to a value considerably larger than its initial value. In the lack of a mechanism able to reduce $|B_z|$ within disks, pregalactic magnetic fields do not help to confine galactic disks. This example clearly illustrates why self-confinement is so rare in nature.

Models based on the confinement by anchored fields to the galactic nucleus will present a similar problem, as $|B_z|$ is expected to decline outwards.

4. THE MAGNETIC ALTERNATIVE

Battaner & Florido (1995, 2000) and Battaner, Florido & Jiménez-Vicente (2002) pointed out that: (1) the magnetic alternative to dark halos does not suffer from the problem of excessive flaring of the H I disk and (2) the existence of truncated stellar disks is explained satisfactorily in the magnetic alternative. Putting aside the problem of radial confinement discussed in the previous sections, it is stimulating to see whether alternative scenarios to conventional ideas have the potential of explaining other phenomena in a natural way. If this occurs, alternative models could attrack much more attention. For instance, some empirical correlations of galactic warps have led to Castro-Rodríguez et al. (2002) to suggest tentatively that outer rotation curves could not be caused by the presence of a massive halo. In the next sections we will show that the flaring problem remains unsolved and that the existence of truncated stellar disks cannot be considered a prediction of the magnetic alternative.

4.1. *The MCP and Implications for the Truncation of Stellar Disks*

If galaxies contained only stars and the observed gas, the initial protodisk of mass M_i should follow approximately a Keplerian rotation, $v_\phi^2 = GM_i/R$, at large R, and evolves to the present-day exponential disk of mass M_f and radial scale length R_d, due to magnetic contraction. The peak velocity of the final disk is $v_{max} = 0.62\sqrt{GM_f/R_d}$ (e.g., Binney & Tremaine 1987). For simplicity, we assume that beyond $2.27R_d$ the rotation curve of a present galaxy is flat with circular velocity v_{max}. During the MCP, a ring of gas located initially at a radius R_i must squeeze up to a final radius R_f, such as $v_{max}R_f = \sqrt{GM_iR_i}$ from angular momentum conservation. Therefore, the initial radius is given by the simple expression $R_i = 0.38(M_f/M_i)(R_f/R_d)R_f$. For a normal spiral galaxy like the Milky Way, with a nearly flat rotation curve up to the last measured point, typically at ~ 30 kpc, and with $R_d \approx 4$ kpc, the MCP should be able to squeeze the disk at least from ~ 85 kpc to ~ 30 kpc in radius, provided the mass loss in outflows is negligible, corresponding to a typical radial velocity of 3 km s^{-1} for one Hubble time. According to §3, this contraction is an unfeasible process due to the global expansive nature of magnetic fields.

The level of contraction may be significantly reduced if a substantial fraction of the mass is lost in winds following early epoch of star bursts, i.e. $M_f < M_i$. Recently, Zhao (2002) has estimated that less than 4% of the total mass of the Galaxy has been lost during its lifetime. Even assuming an amount of mass loss 3 times larger than this value, the magnitude of the contraction of the disk is still huge, from 72 kpc to 30 kpc, again for $R_d = 4$ kpc.

The MCP represents a serious threat to the suggestion by Battaner, Florido, Jiménez-Vicente (2002) that, since stars do not feel the galactic magnetic field, the escape of stars formed beyond a certain galactocentric radius could explain the observed truncation of stellar disks. The calculations by Battaner et al. (2002) are rooted in a scenario that does not consider the MCP, with the gas disks being in anomalous rotation from the very beginning. If one tries to include the MCP in the model, the stellar distribution function depends on different assumptions that concern the details of the MCP, such as the radial velocity of the contraction, or the relationship between star formation rate, surface density and magnetic field strength. Consequently, the escape radius depends on time. If the

disk was contracted from ~ 80 to 30 kpc in radius, it could be even more natural to construct a model in which the stellar surface density decays with R slowly enough, with no stellar truncation radius.

Even assuming that stars in the outer disk were not formed until the gas disk stopped its contraction, which is in contrast with observations that show old stars in the outer disk (e.g., Ferguson & Johnson 2001), the planar velocity dispersions of the stars should increase dramatically with R and this is not observed (e.g., Pfenniger et al. 1994; Vallée 1994).

4.2. *The Flaring Problem*

In the past years, much of the challenge involved in making the magnetic alternative viable concerned the excessive flaring of the disk, which is caused by the vertical magnetic pressure (Cuddeford & Binney 1993). In a two-dimensional analysis, Battaner & Florido (1995) predict a scale height of 3–3.5 kpc at a radius of 15 kpc and of 7 kpc at 30 kpc for the H I disk of M31. These values are still too large by a factor of 3–5 if comparing with the observed H I scale height observed in the Galaxy by Diplas & Savage (1991) and Burton (1992).

This discrepancy in the scale height is the least of our worries. An inspection of the radial profile of the magnetic strength (fig. 3 of Battaner & Florido 1995) reveals that there is something deeply wrong in their calculations. As shown in §2.1, the magnetic field cannot decrease faster than $1/R$ in order to have a disk with a faster rotation. Therefore, a magnetic field of $5\,\mu$G at 15 kpc implies a field strength larger than $2.5\,\mu$G at 30 kpc. The magnetic field estimated by Battaner & Florido (1995, 2000) decays faster than $1/R$.

If the dark matter contribution to the rotation curve is null, the required magnetic fields in the outer parts of typical spiral galaxies are so intense (~ 5–$10\,\mu$G) that the problem of excessive flaring is unresolved. The flaring problem is exacerbated when the magnetic pressure of the small-scale magnetic field is included because it also contributes to the vertical expansion of the disk. This pressure is observed to be comparable to the magnetic pressure of the regular field at least in the optical disk of the Galaxy. On the other hand, the excessive flaring is also aggravated if the magnetic pitch angle is not zero. *We conclude that the flaring problem by itself is sufficient to reject the magnetic alternative.*

5. GALACTIC DISKS WITH SLOWER ROTATION: THE CUSPY HALO PROBLEM

In this Section, we will discuss the role of magnetic fields as a remedy for the cuspy problem of dark halos. It is interesting to see whether the magnetic pressure of the random component, as well as certain configurations of a global-scale magnetic field (not necessarily azimuthal) could help to explain the discrepancy between the profiles of dark halos obtained in cold dark matter (CDM) simulations, which becomes very cuspy towards the center (e.g., Navarro, Frenk & White 1996, 1997), and observations of the halos of dwarf galaxies and LSB, which are better fitted with a core-dominated halo (see de Blok, Bosma & McGaugh 2003, and references therein). One possibility is that galaxies are indeed embedded in cuspy halos follow-

ing the profile found by Navarro, Frenk and White (1996; hereafter NFW) but the combination of asymmetric drifts and magnetic effects wipe out completely the inner cusp, leading to rotation curves that mimic a halo with a core.

The magnetic pressure of the random component gives some support to the disk, reducing the rotation speed. Since it is expected to be in equipartition with the gas turbulent pressure, corrections due to the magnetic pressure should be of the order of the asymmetric drift, ~ 2–3 km s^{-1} (e.g., de Blok & Bosma 2002), which are smaller or comparable to observational uncertainties, typically 4–6 km s^{-1}. This contribution, however, could be larger in a certain radial portion of the disk if the magnetic pressure decays faster than $v_{\rm t}$. Writting the Alfvèn speed associated to the random field as $v_{\rm A}^2 = g(R)v_{\rm t}^2$, with the turbulent velocity dispersion $v_{\rm t}$ constant with R, and $dg/dR < 0$, the rotation curve changes according to the relation:

$$v_\phi^2 = v_{\rm c}^2 + g(R)\frac{v_{\rm t}^2}{2}\left(\frac{d\ln\rho}{d\ln R} + \frac{d\ln g}{d\ln R}\right). \qquad (12)$$

Following de Blok & Bosma (2002), we define $R_{\rm in}$ as the radius of the innermost sampled point of the rotation curve, and assume that at $R > R_{\rm in}$, $g(R) = (R_{\rm in}/R)^n$ for simplicity. Doing so, we are imposing that at $R_{\rm in}$, $v_{\rm A} \approx v_{\rm t}$. The second term of the RHS of Eq. (12) is of the order of the asymmetric drift (comparable to observational uncertainties), whereas the last term is $-nv_{\rm t}^2 R_{\rm in}^n/2R^n$. As the observed rotation curve differs from the NFW profile along a significant range of galactocentric radius, say at $R \gg R_{\rm in} \approx 500$ pc, then $0 < n \leq 1$, implying that $nv_{\rm t}^2 R_{\rm in}^n/2R^n < v_{\rm t}^2/2 \sim 50$ km^2 s^{-2} (for $v_{\rm t} = 10$ km s^{-1}). Consequently, this effect would be only significant very close to the galactic center where $v_{\rm c} \leq 10$ km s^{-1}, a region with large observational uncertainties. Summing up, the contribution from the total pressure to the circular velocity is less than 5–6 km s^{-1}. This result agrees with the fact that CO, Hα and H I rotation curves reveal the same kinematics for the central regions of some galaxies (e.g., Bolatto et al. 2002), which suggests that magnetic fields do not contribute significantly to the support of the gas disks. In particular, the CO rotation curves are expected to trace the motion of the highest density regions, including molecular clouds, which are thought to be hardly affected by the galactic magnetic fields.

In conclusion, magnetic effects are not sufficient to solve the cuspy halo problem satisfactorily.

6. FINAL REMARKS

It has been recognized for decades that the galactic magnetic field provides significant vertical support to the interstellar medium. The role of the magnetic tension in the vertical hydrostatic configuration was considered by Cox (1988) and Boulares & Cox (1990), whereas the role of the tension in the plane of the disk was studied by Nelson (1988), Battaner et al. (1992) and Benjamin (2000) [see also Piddington 1964]. The idea that large-scale magnetic fields can explain the rotation curves of spirals without invoking dark matter has been revived by Battaner & Florido (2000) and Battaner et al. (2002).

All the previous work aimed at giving bounds to magnetic effects on gas dynamics are based on the flaring of the disk by the vertical magnetic gradients. Here we propose

that the radial analysis provides stringent constraints, being free of other complicated assumptions such as the unknown external pressure. We have shown that magnetic fields have a net expansive (or null, but never confining) effect *not only* in the vertical direction but also in the radial direction. Without dark matter halos, the contribution of the magnetic field to the rotation curves at the periphery of spiral galaxies is not expected to be larger than 10 km s^{-1}, which is insufficient to banish dark halos by magnetic fields. This result is valid even though an external pressure, intergalactic magnetic fields or ram pressure are taken into account. In addition, the huge contraction of the disk required in the magnetic alternative would spoil a simple magnetic explanation for the observed truncation of stellar disks. Moreover, the problem of excessive flaring in the outer disk by the magnetic pressure is unresolved and may be aggravated if the random component of the magnetic field is included, or if the field is not purely toroidal. By contrast, there is observational evidence that the dwarf, dark-matter dominated galaxy IC 2574 does not possess a dynamically-important galactic-scale magnetic field. Our first conclusion is that the magnetic alternative is not viable.

Even though there is no doubt about the existence of dark matter halos around galaxies, it is desirable to constrain the magnetic effects on the H I rotation curves for a correct interpretation of the observations. It is suggested that magnetic fields can contribute to produce wiggles in the H I rotation curves, which could be likely associated with non-axisymmetric waves in the disk, as may be the case of NGC 1560. However, the magnetic tension is unlikely to produce substructure in the rotation curve of massive spiral galaxies above 20 km s^{-1}. In Sánchez-Salcedo (1997), the detailed shape of the rotation curve of this dwarf galaxy was fitted reasonably well combining a magnetic field of strength ~ 1 μG plus an isothermal dark halo. Finally, we have considered magnetic fields as a possible solution to the cuspy problem of dark halos. We conclude that they are insufficient to explain the discrepancy between the observations and the predictions of CDM simulations.

APPENDIX

A. WHY AN EXTERNAL PRESSURE IS UNIMPORTANT TO STUDY THE RADIAL CONFIGURATION

Consider a three-dimensional axisymmetric gas disk with surface density $\Sigma(R)$ supported mainly by rotation and embedded in an ambient medium of pressure $P_{\rm ext}$. The transition between the gas disk and the external medium occurs at a vertical height $z_{\rm S}(R)$. From the vertical hydrostatic equilibrium, the radial gradient of the total (turbulent plus magnetic) gas pressure $P_{\rm tot}$ at the midplane of the disk is given by

$$\frac{dP_{\rm tot}(R,0)}{dR} = \frac{d}{dR}\left[P_{\rm ext} + \int_0^{z_{\rm S}(R)} \rho\, \frac{\partial \Phi}{\partial z}\, dz \right] = \frac{d}{dR}\left[\int_0^{z_{\rm S}(R)} \rho\, \frac{\partial \Phi}{\partial z}\, dz \right], \tag{A1}$$

where $\Phi(R,z)$ is the gravitational potential. The pressure exerted by the ambient medium squeezes the disk along z, increasing $P_{\rm tot}(R,0)$ but barely $dP_{\rm tot}(R,0)/dR$, and thereby making the radial confinement extremely difficult. This can be visualized in the following manner. Ignoring for a moment magnetic fields, an external pressure will produce some radial confinement if it is possible to reach a configuration with $\partial P/\partial R\big|_{z=0} > 0$ just by increasing $P_{\rm ext}$:

$$\frac{d}{dR}\left[\int_0^{z_{\rm S}(R)} \rho\, \frac{\partial \Phi}{\partial z}\, dz \right] > 0. \tag{A2}$$

For a gas density decaying along z with a scale height h, for instance $\rho \propto \exp(-z^2/2h^2)$ (for $|z| < z_{\rm S}$), Eq. (A2) is satisfied if the scale height increases with galactocentric radius at least as fast as $h \sim z_{\rm S} \sim R^l \Sigma^{-1}$, where l is an exponent that varies from 2–3 for a gravitational potential with a flat and a Keplerian circular speed, respectively. However, this requirement cannot be fulfilled in approximately isothermal galactic disks with the surface density decreasing outwards, because the maximum variation of h occurs in the limit of large external pressure and goes as $h \sim R^l$.

There are other physical difficulties in the boundaries of the disks if they were confined by $P_{\rm ext}$. From dimensional grounds, it is expected that the gas pressure $P(R,0)$, comprising mainly thermal and turbulent contributions as defined in §2.1, should be greater than the external pressure, as a consequence of turbulent motions and mixing by Rayleigh-Taylor type instabilities in the disk. The total pressure at $z=0$ is

$$P_{\rm tot}(R,0) = P(R,0) + \frac{B_\phi^2(R,0)}{8\pi} > P_{\rm ext} + \frac{B_\phi^2(R,0)}{8\pi}. \tag{A3}$$

In the periphery of the disk, pressure confinement requires $P_{\rm tot}(R,0) \approx P_{\rm ext}$, thus Eq. (A3) implies $B_\phi^2(R,0)/8\pi \ll P_{\rm ext} < P(R,0)$, i.e. well below equipartition and, therefore, they are not sufficient to affect appreciably the rotation curves, according to our discussion in §2.1.

B. DERIVATION OF THE MAGNETIC TERMS IN THE VIRIAL THEOREM

Here we discuss the magnetic terms that appears in the virial theorem. The same derivation can be found in Fiege & Pudritz (2000). Consider a distribution of a electrically conducting plasma with velocity $\boldsymbol{v}$ in a gravitational field $\Phi(R)$

created by all masses in the system. The scalar virial theorem applied to a finite volume V, bounded by a surface S, can be written as:

$$2\mathcal{T} + \mathcal{U} + \mathcal{M}_{\rm cyl} + \mathcal{W} - \int_S \rho\,(\boldsymbol{v} \cdot \boldsymbol{r})(\boldsymbol{v} \cdot d\boldsymbol{S}) = \frac{1}{2}\frac{d^2 I}{dt^2}, \tag{B1}$$

where $\mathcal{T}$ is the kinetic energy of the plasma within V, $\mathcal{W}$ the gravitational term given by $\mathcal{W} = -\int_V \rho\,\boldsymbol{r} \cdot \boldsymbol{\nabla}\Phi\,dV$ and

$$\mathcal{U} = 3\int_V P\,dV - \int_S P\boldsymbol{r} \cdot d\boldsymbol{S}, \tag{B2}$$

$$\mathcal{M} = \frac{1}{8\pi}\int_V B^2\,dV + \frac{1}{4\pi}\int_S (\boldsymbol{r} \cdot \boldsymbol{B})(\boldsymbol{B} \cdot d\boldsymbol{S}) - \frac{1}{8\pi}\int_S B^2 \boldsymbol{r} \cdot d\boldsymbol{S}, \tag{B3}$$

$$I = \int_V \rho\,r^2 dV, \tag{B4}$$

where $\boldsymbol{r}$ is a coordinate vector (e.g., Shu 1992).

In the case of having cylindrical symmetry (i.e., $\partial/\partial z = \partial/\partial\phi = 0$) and for fields with no radial component, $B_R = 0$, a more simple expression for $\mathcal{M}_{\rm cyl}$, which denotes $\mathcal{M}$ under cylindrical symmetry, can be derived when the virial theorem is applied to a finite cylindrical volume; this is done in the following. Since $B_R = 0$, the second term in the RHS of Eq. (B3) contributes only at $z = \pm L/2$ surfaces, where L is the vertical length of the volume V. The component of the field perpendicular to these surfaces is B_z, so that

$$\frac{1}{4\pi}\int_S (\boldsymbol{r} \cdot \boldsymbol{B})(\boldsymbol{B} \cdot d\boldsymbol{S}) = \frac{L}{4\pi}\int_0^{R_S} 2\pi R B_z^2 dR = \frac{1}{4\pi}\int_V B_z^2 dV. \tag{B5}$$

The last term of Eq. (B3) comprises an integral along the vertical surface of the cylinder I_1 and a contribution of the cap surfaces, I_2, given by

$$I_1 = -\frac{1}{4\pi}(B_{\phi,S}^2 + B_{z,S}^2)V, \tag{B6}$$

and

$$I_2 = -\frac{L}{8\pi}\int_0^{R_S} 2\pi R B^2 dR = -\frac{1}{8\pi}\int_V B^2 dV. \tag{B7}$$

Combining the three contributions the final expression for $\mathcal{M}_{\rm cyl}$ reads as

$$\mathcal{M}_{\rm cyl} = \frac{1}{4\pi}\left(\int_V B_z^2\,dV - \left(B_{z,\rm S}^2 + B_{\phi,\rm S}^2\right)V\right). \tag{B8}$$

Similar manipulations allow us to rewrite $\mathcal{U}$ as $\mathcal{U} = 2\int_V P\,dV - 2P_S V$.

For a cold galactic disk, the thermal energy is negligible compared to the kinetic energy. If, in addition, there is no radial accretion flows and the disk can be assumed to be in a steady state, the virial theorem is reduced to

$$2\mathcal{T} + \mathcal{M}_{\rm cyl} + \mathcal{W} = 0. \tag{B9}$$

Assuming that galaxies are isolated systems, $B_{z,\rm S}V$ and $B_{\phi,\rm S}V$ go to zero at large R implying that always $\mathcal{M}_{\rm cyl} \geq 0$, reflecting the impossibility of magnetic confinement in the *radial* direction. In fact, for a given distribution of mass, $\mathcal{W}$ is fixed and if $\mathcal{M}_{\rm cyl}$ is positive, the total kinetic energy should be smaller than in the unmagnetized case in order to satisfy the virial constraint. This does not exclude the possibility that some parts of the disk could present a faster rotation (larger kinetic energy) as long as rotation is slower than that given by gravity in other parts of the disk. It might be that we are observing just the part of faster rotation and the material with a slower rotation lies outside the last point of H I detection, if it is fully ionized by the metagalactic UV background (see Fig. 1 for a visualization). For the case of purely azimuthal fields $\mathcal{M}_{\rm cyl} = 0$, so $2\mathcal{T} + \mathcal{W} = 0$. Therefore, given a distribution of mass, all the configurations in equilibrium must have the same $\mathcal{T}$ regardless of magnetic fields. Note that $\mathcal{M}_{\rm cyl} = 0$ does not imply necessarily that the magnetic configuration is force-free. In the particular case of a force-free magnetic configuration, it can be shown that $\mathcal{M}_{\rm cyl} = 0$ at any arbitrary volume and hence magnetic fields are neither expansive nor confining.

C. CONSTRAINING THE AZIMUTHAL MAGNETIC FIELD STRENGTH IN ISOLATED GALAXIES

Consider an isolated disk galaxy comprising of gas and stars with a given total surface density $\Sigma(R)$. The gas is moving in pure circular orbits (i.e. $v_z = v_R = 0$) and threaded by a purely azimuthal magnetic field. The distribution of mass (stars+gas+dark matter) will create a gravitational potential at $z = 0$, $\Phi_0(R)$. The gravitational circular velocity (not the observed rotation velocity) is defined as $v_c^2 \equiv Rd\Phi_0/dR$. We will assume for simplicity that along the thickness of the H I disk the rotation curve is constant, i.e. $\partial v_\phi/\partial z|_R = 0$. The disk is supposed to be rotating with an anomalous rotation speed $v_\phi(R) > v_c(R)$ out to the radius $R_{\rm kep}$.

We split up the rotational energy, T, in two parts $T = T_0 + \Delta T$, where T_0 is the kinetic energy as if the disk, as it is, were rotating in force balance with the gravitational potential Φ_0 without magnetic fields (keeping all the rest of the

variables of the disk $\Sigma(R)$ and so on unchanged). More specifically, T between two arbitrary radii, say R_1 and R_2, and within $|z| < h_1$, where h_1 is the characteristic scale height of the disk, is given by:

$$T\Big|_{R_1}^{R_2} = T_0\Big|_{R_1}^{R_2} + 2\pi \int_{R_1}^{R_2} \int_0^{h_1} \rho\, R\,(v_\phi^2 - v_c^2)\, dz\, dR, \tag{C1}$$

where $\rho(R, z)$. According to our discussion in Appendix B, $\mathcal{M}_{\rm cyl} = 0$ for pure azimuthal fields, and thus the total rotational energy of the disk must be the same with or without magnetic fields. Therefore, $\Delta T = 0$ between $R = 0$ and ∞, implying

$$\int_0^{R_{\rm kep}} \int_0^{h_1} \rho R\,(v_\phi^2 - v_c^2)\, dz\, dR = -\int_{R_{\rm kep}}^\infty \int_0^{h_1} \rho R\,(v_\phi^2 - v_c^2)\, dz\, dR. \tag{C2}$$

Since we have assumed that $v_\phi(R) > v_c(R)$ at $R < R_{\rm kep}$, this equation implies that $v_\phi(R) < v_c(R)$ outside the $R_{\rm kep}$ circle. Using Eq. (1), the LHS of the equation above reads

$$\int_0^{R_{\rm kep}} \int_0^{h_1} \rho R\,(v_\phi^2 - v_c^2)\, dz\, dR = \frac{1}{8\pi} \int_0^{R_{\rm kep}} \int_0^{h_1} \frac{d(R^2 B_\phi^2)}{dR}\, dz\, dR = \frac{\lambda_B}{8\pi} h_1 R_{\rm kep}^2 B_{\phi,\rm kep}^2, \tag{C3}$$

where $B_{\phi,\rm kep}$ is the field at $R_{\rm kep}$ and λ_B is a dimensionless coefficient less than 1 defined by the relation $\int_0^{h_1} B_\phi^2(R_{\rm kep}, z)\, dz = \lambda_B h_1 B_\phi^2(R_{\rm kep}, 0)$. Analogously, the RHS of Eq. (C2) can be written as:

$$\int_{R_{\rm kep}}^\infty \int_0^{h_1} \rho R\,(v_c^2 - v_\phi^2)\, dz\, dR = \lambda_\rho h_1 \int_{R_{\rm kep}}^\infty \rho(R, 0) R\,(v_c^2 - v_\phi^2)\, dR, \tag{C4}$$

where λ_ρ is a dimensionless parameter less than 1. Thus, the value of $B_{\phi,\rm kep}$ turns out to be

$$B_{\phi,\rm kep}^2 = \frac{8\pi\lambda_{\rho B}}{R_{\rm kep}^2} \int_{R_{\rm kep}}^\infty \rho(R, 0) R\,(v_c^2(R) - v_\phi^2(R))\, dR. \tag{C5}$$

The factor $\lambda_{\rho B} \equiv \lambda_\rho/\lambda_B$ takes into account possible three-dimensional effects. For a disk with a constant scale height with radius, one would expect $\lambda_\rho/\lambda_B \approx 1$, whereas for a disk that flares, it could be a little bit larger, depending on the value adopted for h_1. As $h_1 \to 0$, λ_ρ/λ_B goes to 1, whereas for a value of order of the scale height, $\simeq 400$ pc, $\lambda_\rho/\lambda_B \sim 2$ could be considered as an optimistically high value.

The maximum magnetic effect on the rotation velocity within the $R_{\rm kep}$ circle occurs when $B_{\phi,\rm kep}$ takes its maximum value; this happens when $v_\phi(R) = 0$ for $R > R_{\rm kep}$

$$B_{\phi,\rm kep}^2 \leq B_{\rm cr}^2 \equiv \frac{8\pi\lambda_{\rho B}}{R_{\rm kep}^2} \int_{R_{\rm kep}}^\infty \rho(R, 0) R\, v_c^2(R)\, dR. \tag{C6}$$

$B_{\rm cr}$ is an upper limit to the magnetic field strength at $R_{\rm kep}$.

In the standard dark matter scenario and for galaxies with a flat gravitational circular speed, $v_c(R) = $ constant, the maximum relative contribution of the magnetic field, assuming generously that $B_\phi(R) = B_{\rm cr}$ at $R < R_{\rm kep}$ (see §3.1), is:

$$\frac{v_\phi^2 - v_c^2}{v_c^2}\bigg|_R = 2\lambda_{\rho B} \left(\frac{R_{\rm g}}{R_{\rm kep}}\right) \left(1 + \frac{R_{\rm g}}{R_{\rm kep}}\right) \exp\left(\frac{R - R_{\rm kep}}{R_{\rm g}}\right), \tag{C7}$$

where an exponential law with scalelength $R_{\rm g}$ was adopted for $\rho(R, 0)$ beyond $R_{\rm kep}$.

In the other extreme case that no contribution to the gravitational potential comes from a dark component, $v_c^2(R) \approx GM/R$ at large R, where M is the mass of the luminous matter in the disk. The corresponding maximum relative variation is given by

$$\frac{v_\phi^2 - v_c^2}{v_c^2}\bigg|_R = 2\lambda_{\rho B} \left(\frac{R}{R_{\rm kep}}\right) \left(\frac{R_{\rm g}}{R_{\rm kep}}\right) \exp\left(\frac{R - R_{\rm kep}}{R_{\rm g}}\right). \tag{C8}$$

If the disk is initially, in the lack of magnetic fields, rotating with velocity v_c, the growing magnetic tension during the MCP will produce a contraction of the inner disk and a radial drift of the outer disk. Let us assume that the disk beyond $R_{\rm kep}$ expands in a self-similar fashion and achieves a final density profile $\rho_{\rm f}(R, 0) = (\rho_0/\psi^2) \exp(-R/\psi R_{\rm g})$, with $\psi > 1$ the scale factor. Tthe conservation of angular momentum implies that $v_\phi(R) = \psi^{-1} v_c(R/\psi)$, where $v_c(R/\psi)$ is the circular velocity at R/ψ. In the conventional dark halo scenario, $v_c = $ constant so that $v_\phi(R) = v_c/\psi$. Substituting these relations into Eq. (C5), we get

$$B_{\phi,\rm kep}^2 = \frac{8\pi(\psi^2 - 1)\lambda_{\rho B} v_c^2}{\psi^2 R_{\rm kep}^2} \int_{\psi R_{\rm kep}}^\infty \rho_{\rm f}(R, 0)\, R\, dR. \tag{C9}$$

Therefore, the relative effect on the rotation curves starting with a disk in gravitational force balance is

$$\frac{v_\phi^2 - v_c^2}{v_c^2}\bigg|_R = 2\lambda_{\rho B} \left(\frac{\psi^2 - 1}{\psi^2}\right) \left(\frac{R_{\rm g}}{R_{\rm kep}}\right) \left(1 + \frac{R_{\rm g}}{R_{\rm kep}}\right) \exp\left(\frac{R - R_{\rm kep}}{R_{\rm g}}\right). \tag{C10}$$

Similar manipulations allow us to calculate the same relation but assuming that galaxies do not contain massive dark halos. The corresponding calculation will give

$$\frac{v_\phi^2 - v_c^2}{v_c^2}\bigg|_R = 2\lambda_{\rho B} \left(\frac{\psi - 1}{\psi^2}\right) \left(\frac{R}{R_{\rm kep}}\right) \left(\frac{R_{\rm g}}{R_{\rm kep}}\right) \exp\left(\frac{R - R_{\rm kep}}{R_{\rm g}}\right). \tag{C11}$$

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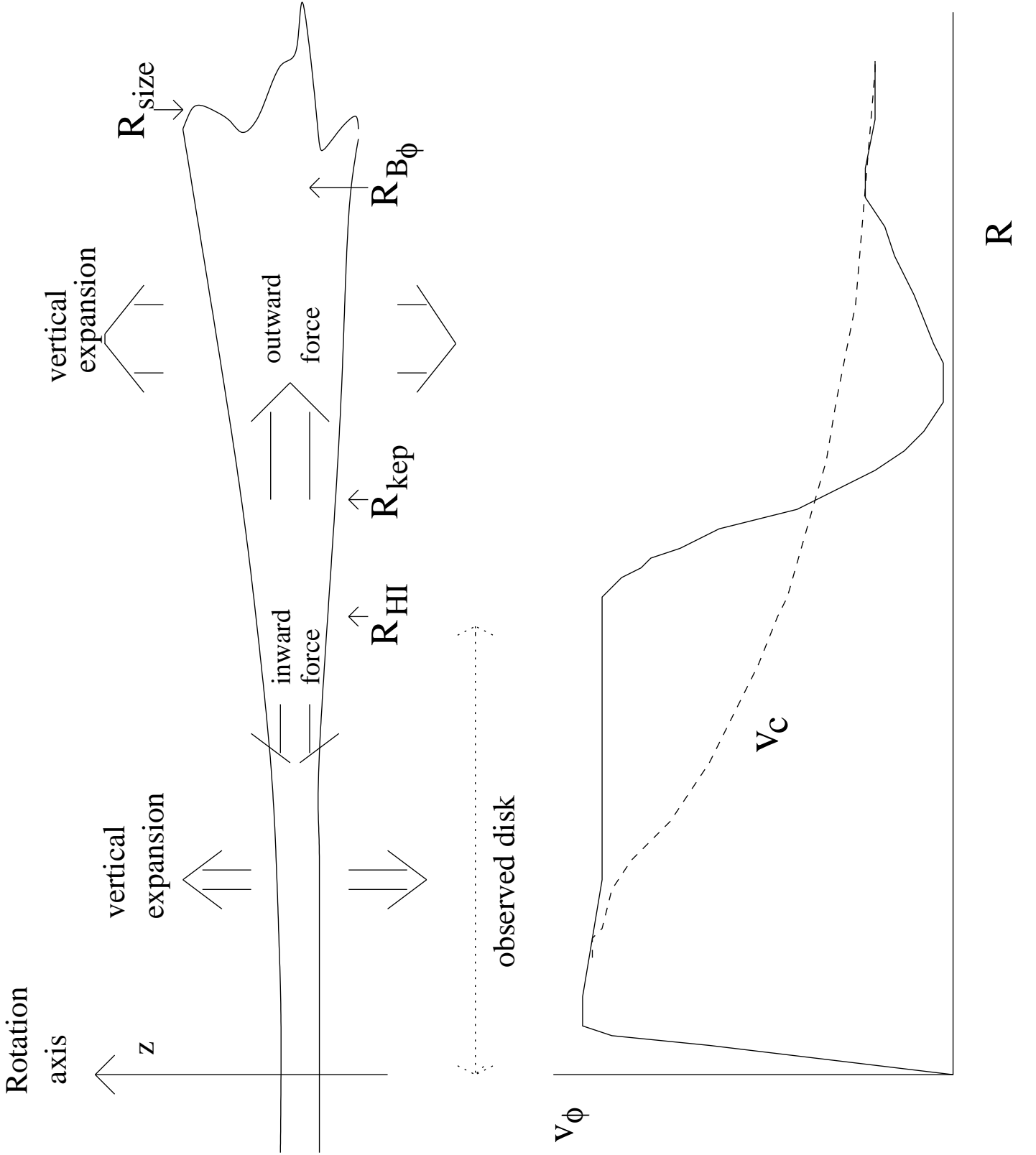
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FIG. C1.— Schematic diagram showing the effect of the magnetic forces in a vertical cross-section of the gas disk in the magnetic alternative (panel a). R_{size} is the radius of the gas disk. Beyond R_{HI}, the outermost radius with H I detection, the gas could be fully ionized. The big arrows indicate the net effect of the magnetic fields onto the gas disk. Beyond R_{kep}, the magnetic forces point outwards in all directions. The expected rotation velocity versus radius is shown in panel b (solid line), together with the gravitational circular rotation speed given by baryonic matter (dashed line). In both panels R has the same scale.